D.(W) Maths
paper 1st, Group-A
Summation of series by
C+iS method

$$= e \quad \underline{|}$$

$$\therefore S = e^{a\cos\theta}\sin(\theta + a\sin\theta)$$

3(ii) $C = \cos\theta + \frac{\cos\theta}{\underline{|1}}\cos 2\theta + \frac{\cos^2\theta}{\underline{|2}}\cos 3\theta + \cdots$

Let $S = \sin\theta + \frac{\cos\theta}{\underline{|1}}\sin 2\theta + \frac{\cos^2\theta}{\underline{|2}}\sin 3\theta + \cdots$

$$\therefore C + iS = (\cos\theta + i\sin\theta) + \frac{\cos\theta}{\underline{|1}}(\cos 2\theta + i\sin 2\theta) + \frac{\cos^2\theta}{\underline{|2}}(\cos 3\theta + i\sin 3\theta) + \cdots$$

$$= e^{i\theta} + \frac{\cos\theta}{\underline{|1}}e^{i2\theta} + \frac{\cos^2\theta}{\underline{|2}}e^{i3\theta} + \cdots$$

$$= e^{i\theta}\left\{1 + \frac{\cos\theta}{\underline{|1}}e^{i\theta} + \frac{\cos^2\theta}{\underline{|2}}e^{i2\theta} + \cdots\right\}$$

Let $x = \cos\theta\, e^{i\theta}$ (1)

$= e^{i\theta}\left\{1 + \frac{x}{\underline{|1}} + \frac{x^2}{\underline{|2}} + \cdots\right\}$

$= e^{i\theta} e^{x} = e^{i\theta} e^{\cos\theta\, e^{i\theta}}$

$= e^{i\theta + \cos\theta(\cos\theta + i\sin\theta)}$

$= e^{i\theta + \cos^2\theta + i\sin\theta\cos\theta}$

$= e^{\cos^2\theta + i(\sin\theta\cos\theta + \theta)}$

$= e^{\cos^2\theta} \times e^{i(\theta + \sin\theta\cos\theta)}$

$\therefore C + iS = e^{\cos^2\theta}\{\cos(\theta + \sin\theta\cos\theta) + i\sin(\theta + \sin\theta\cos\theta)\}$

$\therefore C = e^{\cos^2\theta}\cos(\theta + \sin\theta\cos\theta)$ Ans

3(iii) $S = \sin\theta\cos\theta + \sin 2\theta\, \frac{\cos^2\theta}{\underline{|2}} + \frac{\cos^3\theta\sin 3\theta}{\underline{|3}} + \cdots$

Let $C = 1 + \cos\theta\cos\theta + \frac{\cos^2\theta}{\underline{|2}}\cos 2\theta + \cos^3\theta\, \frac{\cos 3\theta}{\underline{|3}} + \cdots$

and $S = 0 \quad \cos\theta\sin\theta + \frac{\cos^2\theta}{\underline{|2}}\sin 2\theta + \cos^3\theta\, \frac{\sin 3\theta}{\underline{|3}} + \cdots$

$C + iS = 1 + \cos\theta(\cos\theta + i\sin\theta) + \frac{\cos^2\theta}{\underline{|2}}(\cos 2\theta + i\sin 2\theta) + \frac{\cos^3\theta}{\underline{|3}}(\cos 3\theta + i\sin 3\theta) + \cdots$

$= 1 + \cos\theta\, e^{i\theta} + \frac{\cos^2\theta}{\underline{|2}} e^{i2\theta} + \frac{\cos^3\theta}{\underline{|3}} e^{i3\theta} + \cdots$

Let $\cos\theta\, e^{i\theta} = x$

$C + iS = 1 + x + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \cdots$

$C + iS = 1 + x + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \cdots$

$= e^{x} = e^{\cos\theta\, e^{i\theta}} = e^{\cos\theta(\cos\theta + i\sin\theta)}$

$= e^{\cos^2\theta} \times e^{i(\sin\theta\cos\theta)}$

$= e^{\cos^2\theta + i\sin\theta\cos\theta}$

$= e^{\cos^2\theta} \times e^{i(\sin\theta\cos\theta)}$

$C + iS = e^{\cos^2\theta}\{\cos\theta(\sin\theta\cos\theta) + i\sin(\sin\theta\cos\theta)\}$

Equating imaginary part we get (3)

$S = e^{\cos^2\theta}\sin(\sin\theta\cos\theta)$ Ans

(vii) $S = \sin\theta - \frac{\sin 3\theta}{\lfloor 3} + \frac{\sin 5\theta}{\lfloor 5} - \dots$

Let $C = \cos\theta - \frac{\cos 3\theta}{\lfloor 3} + \frac{\cos 5\theta}{\lfloor 5} - \dots$

$\therefore C+iS = (\cos\theta + i\sin\theta) - \frac{1}{\lfloor 3}(\cos 3\theta + i\sin 3\theta) + \frac{1}{\lfloor 5}(\cos 5\theta + i\sin 5\theta) - \dots$

$= e^{i\theta} - \frac{e^{i3\theta}}{\lfloor 3} + \frac{e^{i5\theta}}{\lfloor 5} - \dots$

Let $e^{i\theta} = x$ $\therefore C+iS = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \dots$

$= \sin x = \sin(e^{i\theta}) = \sin(\cos\theta + i\sin\theta)$ [consult $\sin(a+b) = \sin a\cos b + \cos a\sin b$]

$= \sin(\cos\theta)\cos(i\sin\theta) + \cos(\cos\theta)\sin(i\sin\theta)$

$= \sin(\cos\theta)\cosh(\sin\theta) + i\cos(\cos\theta)\sinh(\sin\theta)$

$\therefore S = \cos(\cos\theta)\sinh(\sin\theta)$ Ans

1 (vii) $\sin\theta + \frac{1}{2}\sin 2\theta + \frac{1}{3}\sin 3\theta + \dots$

Let $C = \cos\theta + \frac{1}{2}\cos 2\theta + \frac{1}{3}\cos 3\theta + \dots$

$S = \sin\theta + \frac{1}{2}\sin 2\theta + \frac{1}{3}\sin 3\theta + \dots$

$\therefore C+iS = (\cos\theta + i\sin\theta) + \frac{1}{2}(\cos 2\theta + i\sin 2\theta) + \dots$

$C+iS = e^{i\theta} + \frac{1}{2}e^{i2\theta} + \dots$

$e^{i\theta} = x$, $= x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$

$= -\log(1-x) = -\log(1-e^{i\theta})$

$= -\log(1-\cos\theta - i\sin\theta)$

$= -\log\{2\sin^2\theta/2 - i2\sin\theta/2\cos\theta/2\}$

$= -\log\{2\sin\theta/2(\sin\theta/2 - i\cos\theta/2)\}$

$= -\log 2\sin\theta/2\{\cos(\pi/2 - \theta/2) - i\sin(\pi/2 - \theta/2)\}$

$= -\log 2\sin\theta/2 \cdot e^{-i(\pi/2 - \theta/2)}$

$$C+iS = -\left[\log 2\sin\theta/2 + \log e^{-i(\pi/2-\theta/2)}\right] \quad (3)$$

$$= -\log 2\sin\theta/2 - \log e^{-i(\pi/2-\theta/2)}$$

$$= -\log 2\sin\theta/2 + i(\pi/2-\theta/2)\log e$$

$$\therefore S = (\pi/2-\theta/2), \ \underline{\text{Ans}}$$